Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

Analysis of polarisation change in an electric field-driven liquid crystal lens of cylindrical type where LC are aligned twisted

HyungKi Hong[a]
[a] Department of Visual Optics, Seoul National University of Science and Technology, Seoul, Republic of Korea

Online publication date: 14 June 2011

To cite this Article Hong, HyungKi(2011) 'Analysis of polarisation change in an electric field-driven liquid crystal lens of cylindrical type where LC are aligned twisted', Liquid Crystals, 38: 6, 689 — 696
To link to this Article: DOI: 10.1080/02678292.2011.568639
URL: http://dx.doi.org/10.1080/02678292.2011.568639

Liquid Crystals,
Vol. 38, No. 6, June 2011, 689–696

Taylor & Francis
Taylor & Francis Group

Analysis of polarisation change in an electric field-driven liquid crystal lens of cylindrical type where LC are aligned twisted

HyungKi Hong*

Department of Visual Optics, Seoul National University of Science and Technology, Nowon-gu, Seoul, Republic of Korea

(*Received 14 February 2011; final version received 1 March 2011*)

Electric field-driven liquid crystals (ELC) use the non-uniform distribution of LC directors and show strong dependence on the polarisation direction of the incident light. Phase changes by an ELC lens where LC directors are twisted are investigated. When the polarisation direction of incident light is parallel to the rubbing of the LC cell of the ELC lens on the side of the incident light, the polarisation direction of the propagating light follows the twist of the LC directors. Hence, outgoing light is still linearly polarised and its phase profile can be controlled to match that of a geometric cylindrical lens.

Keywords: autostereoscopic 3D; liquid crystal lens; polarisation; switchable lens

1. Introduction

3D technology is gaining attention in new display applications [1–3]. In the case of autostereoscopic 3D, where no special eyeglass is necessary to perceive the depth, 2D/3D switching is necessary for wider acceptance of 3D display as users still watch the 2D image more often than 3D image. Various 2D/3D switching technologies have been reported, such as changing the liquid crystal (LC) refractive index at the boundary of the lens, or changing the polarisation direction of incident light propagating through an anisotropic lens [4–6]. One of reported switching technologies uses an electric-driven LC lens (ELC lens) where non-uniform distribution of LC directors causes the phase change of the propagating light similar to that of a geometric lens [7]. Figure 1 illustrates the phase change induced by the geometric lens and ELC lens. In the ELC lens, the non-uniform electric fields are caused by multiple electrodes connected to different voltage levels. These electric fields induce non-uniform distribution of the effective refractive index of the LC. Therefore the phase profile of the propagating light is affected, similar to the principle of a Gradient Index (GRIN) lens [8–10]. The typical geometric lens or GRIN lens generally affects the phase of the incident light regardless of the polarisation direction. However, in the ELC lens, the phase change of incident light has a strong dependence on the direction of polarisation.

In the reported ELC structure, only the case where the rubbing direction of LC of homogeneous alignment, polarisation direction and propagating directions of the incident are on the same plane as the y–z plane of Figure 1(b), is analysed [7]. Yet, it often happens that the polarisation direction of the incident light is not on the same plane. In that case, some of the incident light will not be controlled by ELC lens.

In this paper, how the incident light of any polarisation direction can be effectively controlled by ELC lens is investigated. First, the phase change of the propagating light is analysed for the various polarisation directions and the twist of LC directors. The assumption of adiabatic following is used to understand the polarisation change inside the ELC cell. Then the actual phase change by the ELC is calculated and compared with the condition of angles derived from the adiabatic following.

2. Theory

An ELC lens that induces the effect of the cylindrical lens is considered. It is assumed that the incident light propagates along the z-direction and the LC cell is on the x–y plane, as illustrated in Figure 2(a). It is also assumed that thickness of the LC cell of the ELC lens is much smaller than the focal length of the lens, and hence the propagating direction of light is not changed inside the LC cell. The direction of the infinite radius of the lens and electrode of infinite length along the y-direction is assumed as illustrated in Figure 2(a). LC distribution along the y-direction

*Email: hyungki.hong@snut.ac.kr

ISSN 0267-8292 print/ISSN 1366-5855 online

DOI: 10.1080/02678292.2011.568639
http://www.informaworld.com

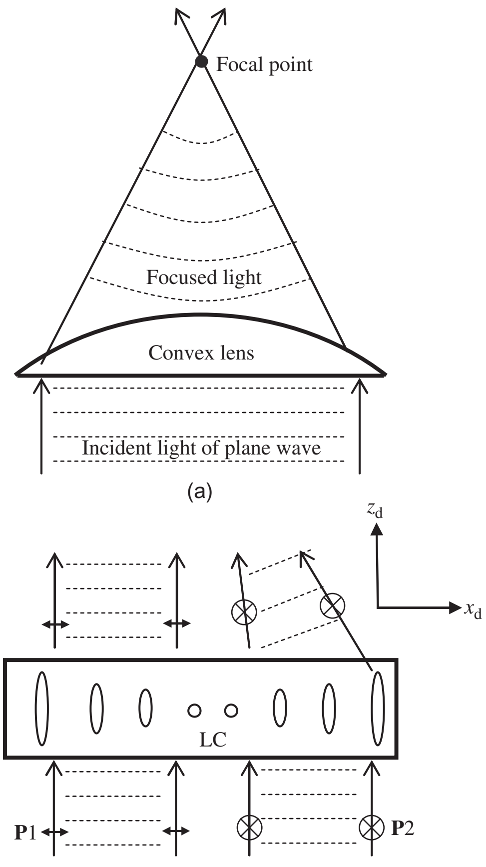

Figure 1. Schematic diagram of phase change induced by (a) the geometric lens and (b) the gradient of refractive index of an ELC lens that is made by non-uniform distribution of LC directors. Dotted lines represent the wave of the same phase. While linear polarisation of y-direction is affected by the non-uniform distribution of LC directors, linear polarisation of x-direction is not affected. **P**1 and **P**2 represent the linear polarisation of x-direction and y-direction, respectively.

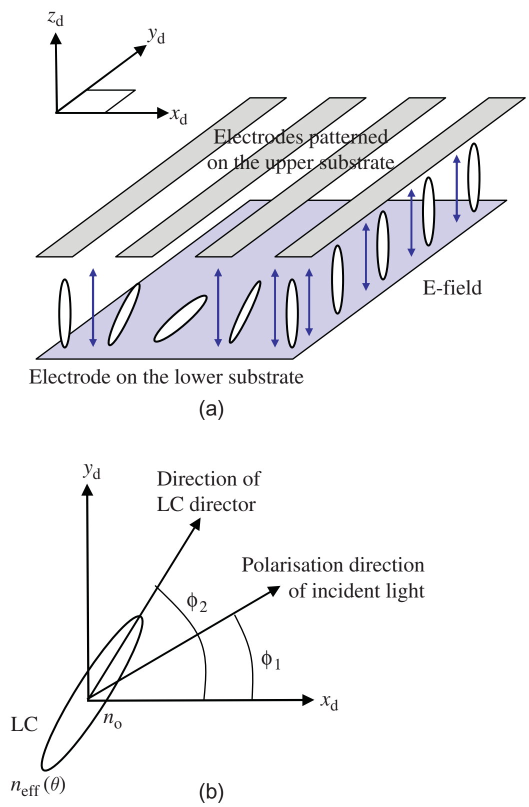

Figure 2. (a) Schematic of an ELC lens of homogeneous alignment. Ellipsoids represent LC directors inside the LC cell. LC directors align homogeneously along the y-direction and rotate to the z-direction by the electric field. (b) Notation of angles of LC directors and the polarisation direction of the incident light. The LC director on the figure is projected on the x–y plane. $n_{eff}(\theta)$ represents the effective refractive index as the function of the polar angle θ.

is not considered. Angles of direction of the LC director and incident polarisation are defined in Figure 2(b). The incident light and the light inside the LC cell can be represented by two linear polarisations **P**1 and **P**2 that are along the x-directions and the y-direction, as illustrated in Figure 3.

When the LC cell is considered to consist of meshes of finite size, the phase at the position (x_m, z_n) inside the LC cell is determined by the direction of the LC directors at position (x_m, z_n) [11, 12]. Each mesh can be treated as one uniform uniaxial medium. Phase change at the position (x_m, z_n) can be written in Jones's Matrix Representation as:

$$M(x_m, z_n) = R(-\phi_2(x_m, z_n))P(x_m, z_n)R(\phi_2(x_m, z_n)). \tag{1}$$

Rotation matrix $\boldsymbol{R}$ and propagation matrix $\boldsymbol{P}$ are given as:

$$R(\phi_2) = \begin{pmatrix} \cos\phi_2 & \sin\phi_2 \\ -\sin\phi_2 & \cos\phi_2 \end{pmatrix}, P(x_m, z_n) = \begin{pmatrix} \exp\left(-i\frac{2\pi\,\Delta z\cdot n_o}{\lambda}\right) & 0 \\ 0 & \exp\left(-i\frac{2\pi\,\Delta z\cdot n_{eff}(x_m,z_n)}{\lambda}\right) \end{pmatrix}. \tag{2}$$

λ and Δz represent the wavelength and size of mesh along the z-direction. Then, overall phase change that occurs as light passes through z_n layers can be written as:

$$M(x_m) = \sum_{z_n=0}^{z_N} M(x_m, z_n). \tag{3}$$

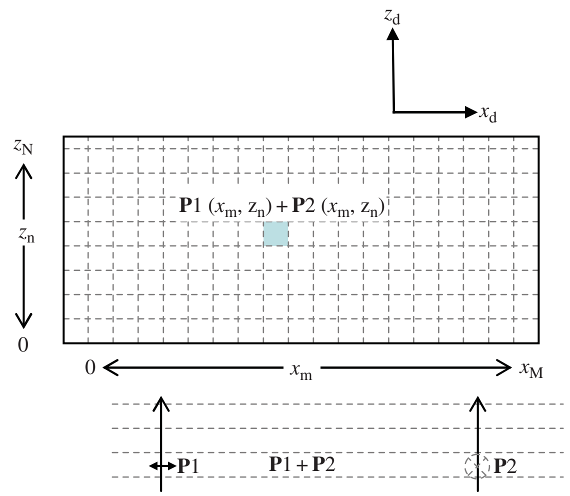

Figure 3. Polarisation state inside the LC lens for the incident plane wave that propagates along the z-direction. **P**1 and **P**2 represent the linear polarisation of x-direction and y-direction, respectively. x_M and z_N represent the lens pitch and the thickness of the LC cell.

When LC directors align homogeneously along the y-direction, ϕ_2 is independent of the position (x_m, z_n) and Equation (3) can be written as:

$$M(x_m) = R(-\phi_2) \sum_{z_n=0}^{z_N} P(x_m, z_n) R(\phi_2)$$

$$= R(-\phi_2) \begin{pmatrix} \exp\left(-i\frac{2\pi \cdot z_N \cdot n_o}{\lambda}\right) & 0 \\ 0 & \exp\left(-i\frac{2\pi\,\Delta z \sum_{z_n} n_{eff}(x_m, z_n)}{\lambda}\right) \end{pmatrix} R(\phi_2). \quad (4)$$

Unit vectors of **P**1 and **P**2 are (1 0) and (0 1) in Jones's Matrix Representation. Equation (4) shows that the phase change for the **P**1 component of the incident light is the same irrespective of position x_m. Local distribution of LC directors will only affect the **P**2 component. This means that the ELC lens can control the propagating direction of **P**2 but not **P**1 for the LC cell of homogeneous alignment.

Examples of various autostereoscopic 3D are illustrated in Figure 4. A lenticular lens consists of a multitude of cylindrical lenses. The direction of the lenticular lens is defined as the direction of the infinite focal length. In the case of an ELC lens using an LC cell of homogeneous alignment, the direction of a lenticular lens is parallel to the LC alignment direction. Figure 4(a) illustrates the case where the directions of the lenticular lens and the incident polarisations are along the y-direction. Incident light consists of only **P**2 polarisation. If an ELC lens is used as the lenticular lens, the propagating direction of the incident light can be completely controlled by the ELC lens, and the phase change by ELC lens is determined by Equation (4).

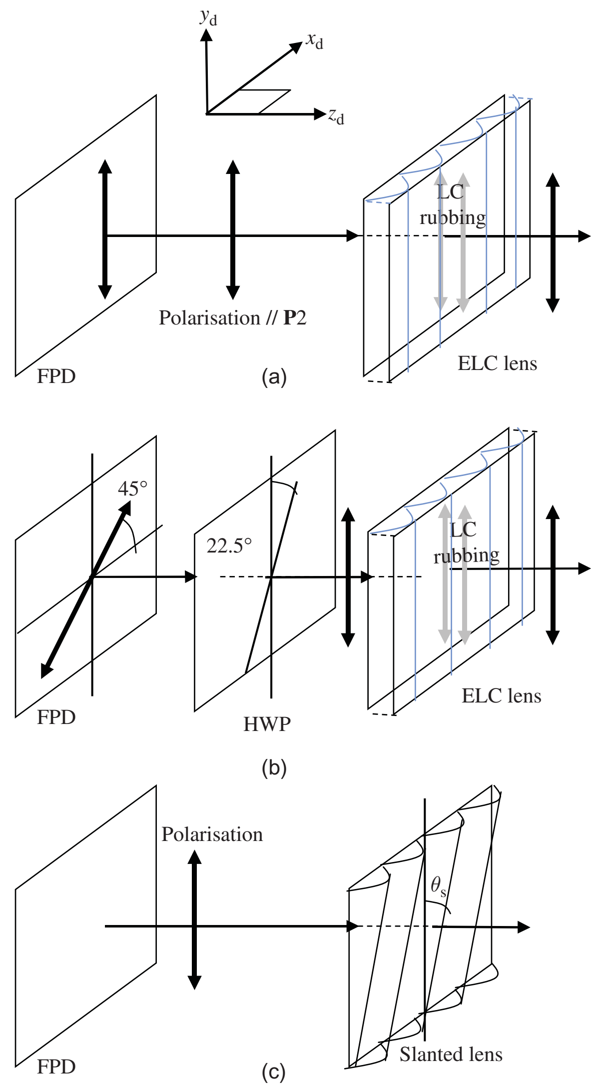

Figure 4. Examples of autostereoscopic 3D. (a) 2D /3D switching displays where the polarisation direction of the incident light and the direction of the lenticular lens are vertical. (b) 2D /3D switching displays where polarisation direction of the incident light is 45° and the direction of the lenticular lens is vertical. (c) Autostereoscopic 3D where the polarisation direction of the incident light is vertical and the direction of the lenticular lens is slanted. The direction of the lenticular lens is defined as the direction of the infinite focal length. FPD represents Flat Panel Display. Double-sided grey and black arrows represent the direction of the rubbing and the polarisation.

Figure 4(b) illustrates the case where the polarisation direction on the x–y plane is 45° and the direction of the lenticular lens is 90°. A liquid crystal display (LCD) of twisted nematic (TN) mode is one example of a FPD (flat panel display) that emits such polarised light. Light coming from the FPD consists of an equal amount of **P**1 and **P**2 polarisation. If this light is directly incident on the ELC lens, the **P**1 component will not be affected by the ELC lens and cause the degradation of 3D display performance. To prevent such degradation, a half wave plate (HWP) was placed between the FPD and ELC lens in Figure 4(b) [7]. The HWP converts the polarisation direction from 45° to 90°. The other case where the polarisation direction is not parallel to the direction of the lenticular lens is in an autostereoscopic multiview display, where a slanted lenticular lens is used as in Figure 4(c) [13, 14]. When a suitable slanted angle of the lenticular lens is selected, RGB subpixels in the observed 3D image are reported to be uniformly distributed.

As the polarisation direction of the incident angle of an LCD is typically one of 0°, 45° and 90°, the polarisation direction of the incident light is not parallel to the direction of the lenticular lens as well as the LC direction of the ELC lens of homogeneous alignment. Therefore light incident on the ELC lens can be decomposed into **P**1 and **P**2 components. The **P**1 component of the light leaving the ELC lens will not match the desired light distribution, as its phase profile is different from that of the lens profile. These examples show the limitation of the reported ELC lens structure of homogeneous alignment.

On the other hand, Equation (3) shows that the phase and amplitude of **P**1 and **P**2 polarisations can be controlled when the twist of the LC directors is not zero. Thus, phase change by the twisted LC cell is considered. Phase retardation Γ at each position is defined as $\Gamma = 2\pi \cdot z_N \left(n_{eff}(x_m) - n_o\right)/\lambda$. When the change of the LC twist is sufficiently small compared with the phase retardation Γ, and the polarisation direction of the incident light is parallel or perpendicular to the LC director on the side of incident direction, the adiabatic following is known to happen [11, 12]. When the adiabatic following occurs, the polarisation direction of the propagating light follows the twist of LC directors, and the polarisation direction of the output light will be parallel or perpendicular to the rubbing direction of the LC on the opposite side of the incident direction. In an ELC lens, if the rubbing direction of LC on the side of one substrate is selected to be parallel to the polarisation direction of the incident light, and the assumption of the adiabatic following is valid, the polarisation direction of the outgoing light will be parallel to the rubbing direction of the other substrate. If the rubbing direction of the other substrate is selected as the y-direction, the outgoing light will consist only of one polarisation parallel to the LC director on

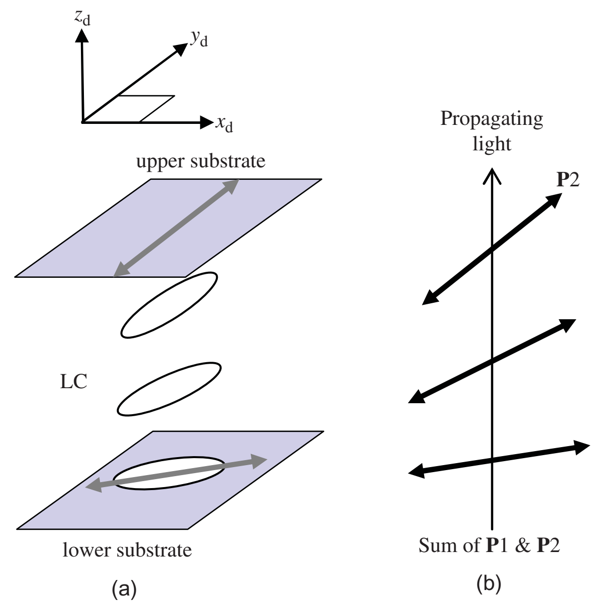

Figure 5. Adiabatic following in the ELC lens. (a) Alignment of LC directors. Double-sided grey arrows represent the direction of the rubbing. (b) Polarisation direction as light propagates through the LC cell. Double-sided black arrows represent the direction of the polarisation.

the opposite side of the incident direction, and the phase change of this polarisation can be completely controlled by the ELC lens. This is illustrated in Figure 5.

If the polarisation direction of the incident light is not parallel to the LC director on the side of incident direction, it can be decomposed into two polarisations that are parallel and perpendicular to the LC director. These two polarisations will follow the twist of the LC directors, respectively. However, the effective refractive index for the polarisation perpendicular to the LC director is always the refractive index of ordinary wave, n_o, irrespective of the direction distributions of the LC directors. Thus the phase profile of the polarisation perpendicular to the LC director will not match the phase profile of the geometric lens.

In the ELC lens, the LC directors do not twist uniformly with the same polar angle θ at the position x_m, as the vertical and horizontal components of electric field are non-uniform. Thus the actual phases at each position of the ELC lens are calculated to confirm the validity of the adiabatic following.

3. Simulation

The designed viewing distance of autostereoscopic 2-view display is determined as $f(1 + 2\ IOD/P_{lens})$ where f is the focal length, IOD is the interocular distance between the eyes and P_{lens} is lens pitch [2].

As an example of lens design, an imaging display of RGB subpixel configuration of vertical stripe type and subpixel pitch of around 0.1 mm is considered. Lenticular lens condition is selected such that the focal length f is 1.7 mm and pitch of each cylindrical lens is 0.2 mm. This corresponds to the condition of autostereoscopic 2-view display with the designed viewing distance of 1100 cm.

Material parameters of LC used for the simulation are that the dielectric constant $\varepsilon_e = 8.3$ and $\varepsilon_o = 3$, refractive index $n_e = 1.5977$ and $n_o = 1.4828$, elastic constant $K_{11} = 13$ pN, $K_{22} = 5.8$ pN, $K_{33} = 12.7$ pN. The phase change caused by a geometric lens of the focal length f and lens pitch x_M can be written as:

$$\Delta\phi = k \cdot x_d^2/2f, 0 < x_d < x_M/2. \tag{4}$$

x_d is the distance from the centre of lenticular lens along the x-direction. The phase change by the ELC lens should be larger than the maximum value of Equation (4). The thickness of the LC cell is selected as 50 μm. As the thickness of the LC cell of the ELC lens is much smaller than the focal length of the lens, it is assumed that propagating direction of the light is not changed inside the LC cell.

Figure 6 illustrates the ELC structure used for the simulation. Commercial software (Techwiz 2D) is used to calculate the motion of LC directors under the electric field [15]. Infinite length is assumed along the y-direction and the periodic boundary condition is used. The electrode on the side of the lower substrate is not patterned and connected to the ground voltage. Twenty electrodes of 5 μm width are located with a distance of 5 μm on the side of the upper substrate and connected to the different voltage levels. If the rubbing direction on that side of the patterned electrodes is not 90°, domains may be generated by the horizontal fringe fields. The fringe field of horizontal direction is weaker on the lower side than the upper side where multiple patterned electrodes exist. Hence, the rubbing direction on the side of the upper substrate is fixed as 90° and the rubbing direction of the side of the lower substrate is selected as the simulation parameter.

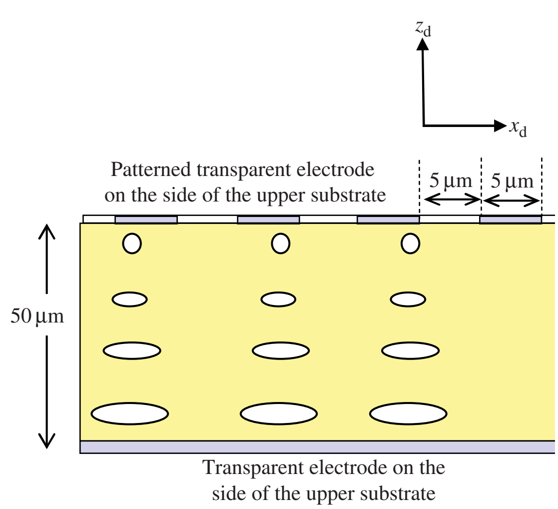

Figure 6. Configuration of the ELC lens structure for the simulation. The electrode on the side of the lower substrate is not patterned. Electrodes on the side of the upper substrate are patterned with the interval of 5 μm and width of 5 μm, and connected to the different voltage levels (colour version online).

Examples where the polarisation direction of the incident angle and the direction of the lenticular lens are different are illustrated in Figure 4(b) and 4(c). Performance of twist angles of 45° and 9.54° are calculated and compared with a LC cell of homogeneous alignment. A slanted angle of 9.54° is selected, as the spatial sub-pixel distribution of each viewing zone was reported to be uniformly distributed for autostereoscopic 9-view 3D display using a lenticular lens at this slanted angle [13, 14].

Once distribution of the LC directors under various driving voltages is calculated, polarisation states at position x_m are calculated using Equations (1) and (3). The polarisation direction ϕ_1 of incident light is used as the other parameter.

4. Results and analysis

Figure 7 illustrates the calculated result for the ELC lens of the homogeneous alignment. Figure 7(a) shows LC directors and the contour line of electric field distribution. Figure 7(b) shows that the normalised intensity of **P**2 for the polarisation direction of the incident light, $\phi_1 = 45°, 60°, 90°$. The normalised intensity of **P**2 decreases as the polarisation direction of the incident light, ϕ_1, deviates from 90°. Figure 7(c) shows the relative phase profile of the geometric lens, **P**1 and **P**2 polarisation of light leaving the ELC lens for the polarisation direction of the incident light, $\phi_1 = 90°$. While the phase profile of **P**2 is similar to that of the geometric lens, the phase profile of **P**1 is quite different from that of the geometric lens at the same driving voltage levels. Phase profiles of **P**2 at the polarisation direction of the incident light, $\phi_1 = 45°, 60°$ are calculated to be the same as that of **P**2 at $\phi_1 = 90°$. So, if the incident light consists of **P**1 and **P**2 components, only the phase of **P**2 can be controlled to match the lens profile by the ELC lens of the homogeneous alignment.

Figure 8 illustrates the calculated result of the distribution of the LC director and the normalised

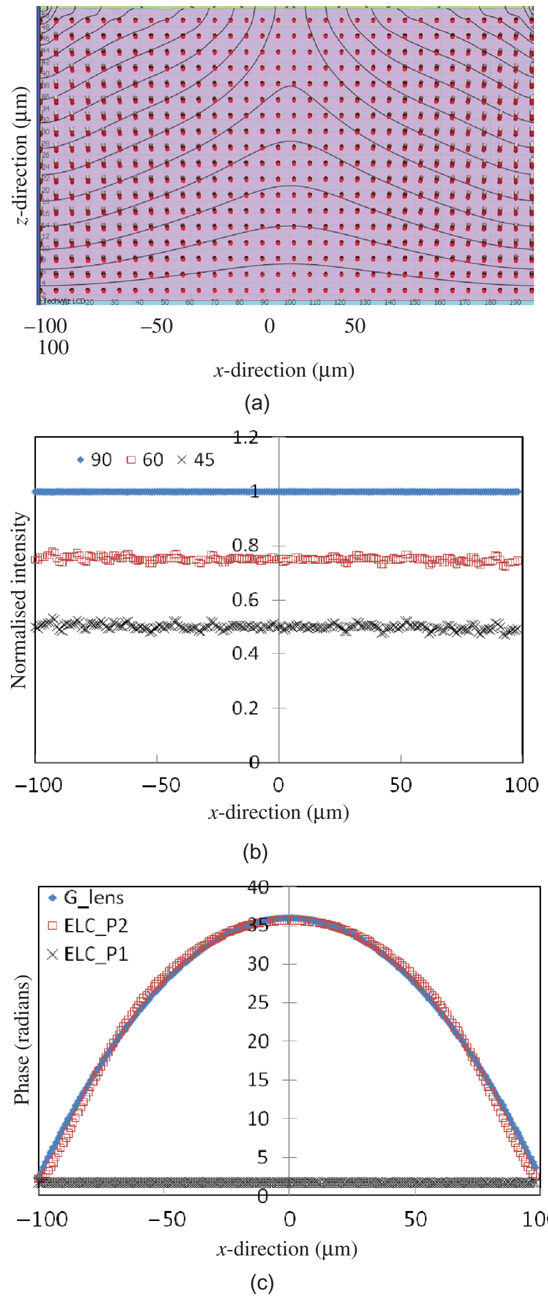

Figure 7. Calculated result for the ELC lens cell of homogeneous alignment. (a) Distribution of LC directors under electric field. (b) Normalised intensity of **P**2 where the polarisation directions of the incident light, ϕ_1 are 45, 60 and 90°. Numbers on the upper left side represent the condition of ϕ_1. (c) Phase profile of the geometric lens and **P**1 and **P**2 for the polarisation direction $\phi_1 = 90°$ (colour version online).

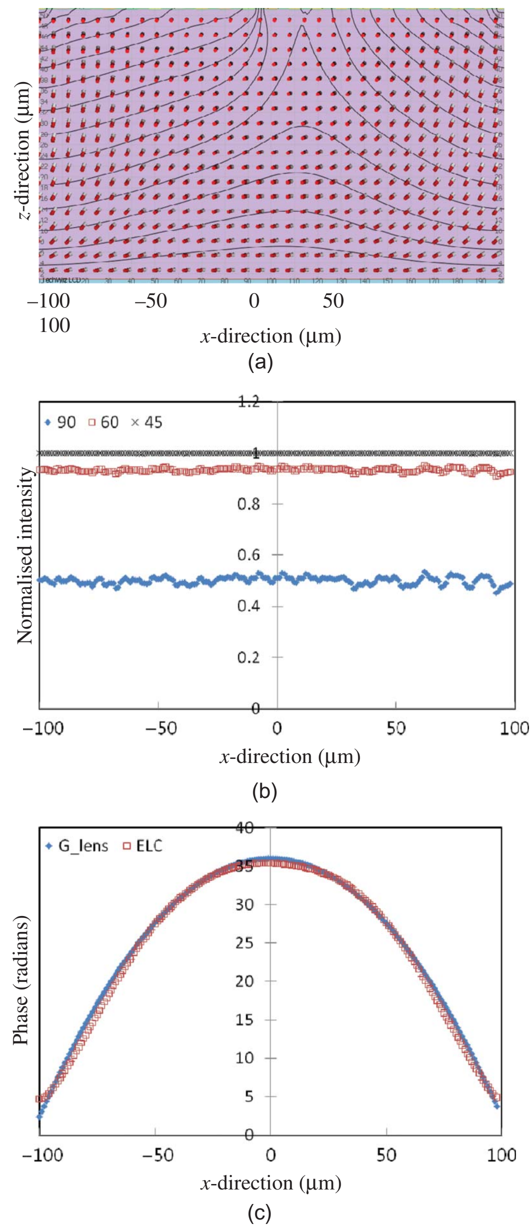

Figure 8. Calculated result for the ELC lens cell of 45° twist. (a) Distribution of LC directors under electric field. (b) Normalised intensity of **P**2 where the polarisation directions of the incident light, ϕ_1 are 45, 60, 90°. Numbers on the upper left side represent the condition of ϕ_1. (c) Phase profile of the geometric lens and **P**2 for $\phi_1 = 45°$ (colour version online).

intensity of the **P**2 component of the light leaving the ELC lens of 45° twist. In Figure 8(a), LC directors at the left side should be driven by the larger driving voltage levels compared with those at the right side, to obtain the symmetric phase profile. This can be attributed to the interaction between the twist direction of LC directors and the horizontal driving field. Figure 8(b) shows that the normalised intensity of **P**2 is 1 at the polarisation direction of the incident light, $\phi_1 = 45°$, and decreases as ϕ_1 deviates from 45°. Intensities at $\phi_1 = 0°$ and 30° are similar to those of 90° and 60°, respectively. This means that linear polarisation follows the LC twist as predicted by the assumption of adiabatic following. These calculated results show that the assumption of the adiabatic following is still valid for the ELC lens where distributions of LC directors are not uniform. Hence, phase change of lens profile can still be obtained even when the polarisation direction of the incident light is not along the *y*-direction. If this twisted structure is applied to that of Figure 4(b), HWP is no longer necessary.

Figure 9 shows the contour map of the azimuth angle of LC directors for the above two conditions. A horizontal electric field induces the azimuth motion of LC directors for these two conditions. They show that the azimuth angle along the *z*-direction is not uniformly changed.

Table 1 shows examples of voltage levels applied to the patterned electrodes to make phase profiles of the ELC lens similar to that of the geometric lens. While voltage levels of ELC of homogeneous alignment are symmetric with respect to the 11th electrode, those of ELC of 45° twist are not symmetric.

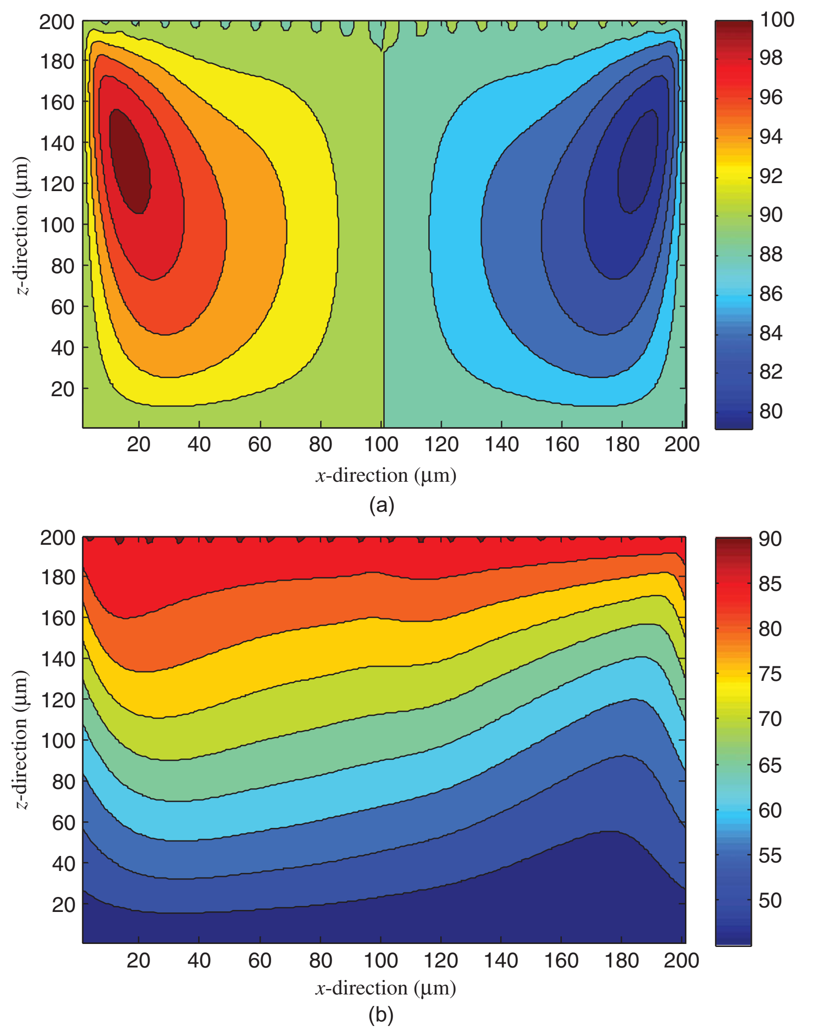

Figure 9. Contour map of the azimuth angle of LC directors under the electric field for the ELC lens cell of (a) homogenous alignment and (b) 45° twist.

Table 1. Examples of voltage levels applied to the electrodes on the side of substrate when the phase profile of ELC lens is similar to that of geometric lens. The 11th electrode corresponds to the centre of the cylindrical lens.

Electrodes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0 twist	3.3	2.7	2.5	2.3	2.2	2.1	2.0	1.8	1.6	1.3	1.0	1.3	1.6	1.8	2.0	2.1	2.2	2.3	2.5	2.7
45 twist	3.0	2.7	2.6	2.5	2.4	2.3	2.2	2.15	2.1	2.0	1.0	1.0	1.0	1.3	1.6	1.8	2.0	2.2	2.4	2.6

In the case of a slanted lens of slanted angle ϕ_s, the intensity of the **P**2 component will be approximately $\cos^2(90 - \phi_s)$ for an ELC cell of homogeneous alignment. For example, if $\phi_s = 9.54°$, the normalised intensity of **P**2 and **P**1 will be about 0.97 and 0.03, and the propagating direction of **P**1 polarisation will be different from the designed distribution. The ratio between the unwanted light distribution and the designed one is generally defined as 3D crosstalk. It is reported that a few percent of crosstalk can be observed and cause degradation of image quality [16]. Depending on the value of the slanted angle, it may worthwhile considering the twisted configuration to reduce the **P**1 component of the outgoing light. Figure 10 illustrates autostereoscopic 3D where the polarisation direction of the incident light and the lenticular lens direction are different. In Figure 10(b), the coordinate system is rotated with respect to the z-axis such that the y-axis is parallel to the direction of the slanted lenticular lens. By applying the proposed twisted ELC lens, linear polarisation of a designed phase profile can be obtained and the unwanted light distribution can be reduced.

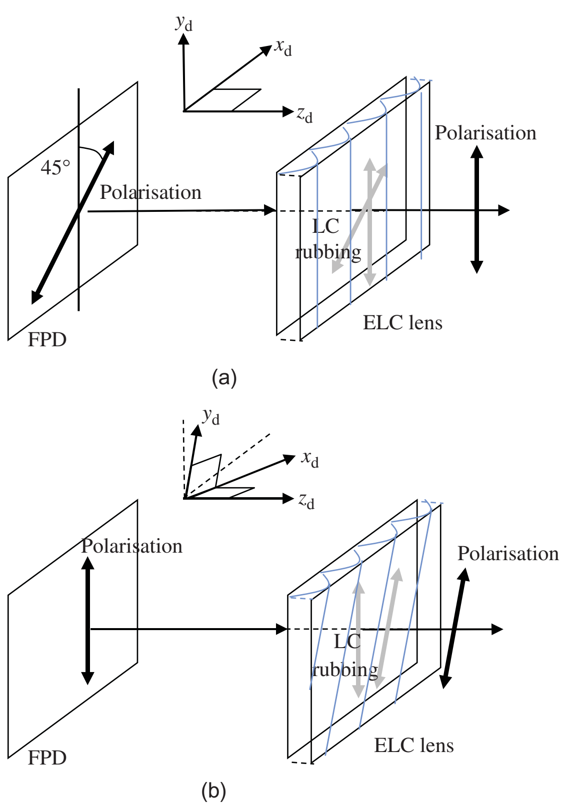

Figure 10. Configuration of ELC. (a) LC of 45° twist is used for the polarisation direction of $\phi_1 = 45°$. (b) Patterned electrodes on the side of the right substrates are slanted. Coordinate system is changed to match the configuration of the slanted lens. Double-sided grey and black arrows represent the direction of the rubbing and the polarisation.

5. Conclusion

In an ELC lens where the LC is aligned with the non-zero twist angle, the phase changes for each polarised light have been analysed. When the polarisation direction of the incident light is different from lenticular lens direction, phase of the leaving light can be made to match the phase of the geometric lens by selecting the rubbing direction on the lower substrate parallel to the polarisation direction of the incident light.

The proposed ELC lens structure of twisted LC directors will be effective in making a simplified configuration compared with the reported structure where HWP was added to rotate the polarisation direction or to reduce the amount of light whose propagating direction is not correctly controlled.

Acknowledgement

The author thanks SANAYI System Co., Ltd for help with TechWiz 2D Software.

References

[1] Okoshi, T. *Three Dimensional Images Techniques*; Academic Press: New York, 1976.
[2] Javidi, B.; Okano, F. *Three-Dimensional Television, Video, and Display Technologies;* Springer: Berlin, 2002.
[3] Son, J.Y.; Javidi, B. *J. Display Tech*. **2005**, *1*, 125–140.
[4] Woodgate, G.J.; Harrold. J. *J. Soc. Inform. Display* **2006**, *14*, 421–426.
[5] Krijn, M.P.C.M.; de Zwart, S.T.; de Boer, D.K.G.; Willemsen, O.H.; Sluijter, M. *JSID* **2008**, *16*, 847–855.
[6] Ren, H.; Fox, D.W.; Wu, S.T. *Opt. Express* **2007**, 1328–1335.
[7] Hong, H.K.; Jung, S.M.; Lee, B.J.; Shin, H.H. *J. Soc. Inform. Display* **2009**, *17*, 399–406.
[8] Kowel, S.T.; Cleverly, D.S.; Kornriech, P.G.; *Appl. Opt*. **1984**, *23*, 278–289.
[9] Konforti, N.; Marom, E.; Wu, S.T. *Opt. Lett.* **1988**, *13*, 251–253.
[10] Ren, H.; Fan, Y.H.; Gauza, S.; Wu, S.T. *Jpn. J. Appl. Phys.* **2004**, *43*, 652–653.
[11] Yeh, P.; Gu, C. *Optics of Liquid Crystal Displays*; John Wiley & Sons: Chichester, 1999.
[12] Yang, D.K.; Wu, S.T. *Fundamentals of Liquid Crystal Devices*; John Wiley & Sons: Chichester, 2006.
[13] van Berkel, C. *Proc. SPIE* **1999**, *3639*, 84–91.
[14] Hong, H.K.; Park, J.; Lee, S.C.; Shin, H.H. *Displays* **2009**, *29*, 512–520.
[15] SANAYI System Co., Ltd, http://www.sanayisystem.com (accessed Feb 2011).
[16] Patterson, R. *J. Soc. Inform. Display* **2009**, *17*, 987–996.